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On the Shadow Wage of Labor: An Extension of Some Recent Theoretical Contributions

by

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The literature on development economics and project evaluation in developing countries emphasizes that in a perfectly competitive fully employed economy the cost to the economy of employing an additional unskilled worker on a project is the output foregone elsewhere by such employment;[1] in the absence of externalities or monopoly-monopsony elements in product and labor markets, this cost is adequately expressed by the project wage. However, if there is open unemployment of unskilled labor (i.e. workers without jobs who are demonstrably seeking employment), employment of an additional worker on a project may lead to a reduction in unemployment else where in which case output foregone is zero and the project wage overstates the cost to the economy which is supposedly zero.[2] Moreover, it is widely held that in some parts of Asia (at least during some parts of the year) the marginal product of labor in agriculture is zero -- i.e. even though labor may be visibly employed the withdrawal of marginal workers would not lead to a fall in total agricultural production.[3] In this case, any wage paid to such labor in alternative employment would overstate the cost to society which is again zero. Clearly the argument of a zero marginal product or a marginal product below the going wage rate should not be applied to the case of those farms which are actually hiring outside farm help, for no rural household would knowingly

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1 The emphasis in this paper is on unskilled workers by which term we shall take to mean those labors whose jobs require little or no formal education or on-the-job training, the basic skills required being obtainable easily within a few days or weeks on the job. Note that while this paper focuses on unskilled labor, there is no reason why the logic of the argument could not be extended to semi-skilled and skilled laborers should the phenomena of unemployment and under-employment pertain to them as well.

2 Note that for the conclusion that output foregone is zero to hold, it is not necessary that the worker hired for the project would himself have been unemployed in the absence of the project. The same conclusion would hold if the hiring of a previously employed worker led to his being replaced by an unemployed worker and by extension to the case where the ultimate reduction in unemployment is even further removed.

3 It is common to identify output foregone with the marginal productivity of the withdrawn laborer. Strictly speaking this is correct only for hired workers; in the case of family labor, output foregone will be the same as the marginal productivity of the withdrawn worker only on the additional assumption that the work effort of the remaining family members remains constant. Normally one would expect output foregone would be less than the marginal product of the withdrawn worker. Once one grasps this basic point, it is easy to imagine the possibility that output foregone could be zero even in the event that the marginal product of the withdrawn worker is positive. See A. K. Sen, "Peasants and Dualism With and Without Surplus Labor", Journal of Political Economy, Vol. LXXIV, No. 5 (October, 1966), pp. 425-450.

expand the number of workers hired beyond the point where the marginal product of labor equals the real wage since to do so would reduce the level of household welfare. However when it comes to decisions about the use of family owned labor, the usual profit-maximizing assumptions may not hold; decisions about the application of family labor may be determined more by tradition and custom than by the marginal calculus emphasized by economists (e.g. families may decide to maximize output within the constraint of their labor inputs available and each worker's share in total output may be determined more by his average product than his marginal product). In this event, the going wage rate for agricultural workers or for local industries may not reflect the product foregone from withdrawing labor from such farm households; these households may not supply labor even when wages exceed the product gained from employing workers on the farm because of lack of knowledge that such employment opportunities exist, a dislike of wage work as opposed to self-employment, and a fear by family members that by working elsewhere they may lose their claim to a share in family output or to a share in the inheritance of the family land, etc. Even if such households do supply some wage labor, the wage they are paid may not reflect output foregone for the reasons cited above. Thus, the phenomenon of "disguised unemployment" or "underemployment" (as opposed to open unemployment) is held to apply in any case where the marginal product of labor (in alternative employment) lies below going wage rates, regardless of whether that marginal product of labor is in fact zero or not. In this case, the market wage rate no longer reflects the true costs to society from employing such workers.

The common thread to all the arguments above is the idea that the real cost of employing an additional worker on a project is the output foregone. This idea provides the basis for one part of the shadow wage rate in recent project evaluation studies for UNIDO by A. K. Sen; Stephen Marglin; Partha Das Gupta and for O.E.C.D. by I.M.D. Little and James Mirlees.[4] The output foregone elsewhere from employing an additional worker on project only measures the direct costs to society however. Both studies stress that where the level of aggregate savings is not optimal, the decision to employ an additional worker may have an effect on the mix of aggregate consumption and savings imposing an additional indirect cost on society which must be included in the calculation. Assuming that the market cost, W, of an additional worker employed on the

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[4] See UNIDO, Guidelines for Project Evaluation, (New York: United Nations Publication, 1972), Ch. 15 and I.M.D. Little and James Mirlees, Manual of Industrial Project Analysis in Developing Countries, Vol II (Paris: Development Centre of O.E.C.D., 1968), Ch. XIII.

project is financed by a tax of W and that workers consume all of their incomes, the shadow wage rate, W*, becomes

$$(1) \quad W^* = Z + [(1-S^T) + S^T P^{INV}] W - W$$

$$= Z + S^T (P^{INV} - 1) W$$

where Z is the alternative output foregone, S^T is the marginal propensity to save of taxpayers, and P^{INV} is the shadow price of savings.[5] This is the UNIDO formula for the shadow wage rate; it measures the aggregate consumption costs to society stemming from the employment of an additional worker.[6] Looking at the first expression for W* (before simplification), Z is foregone output measured at market prices which correctly reflects society's aggregate consumption costs because of the assumption that workers (and their families) consume their entire incomes; the second term expresses the fact that a tax of W causes taxpayers consumption and savings to fall by $(1-S^T)$ W and $S^T W$ respectively and the latter term must be multiplied by the shadow price of savings to measure the aggregate consumption costs to society; the last term W measures the gains to workers (and their families) from the receipt of the wage and

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5 Where aggregate savings are suboptimal, the value of P^{INV} will be greater than one. The reason is that suboptimal savings imply that the marginal rate of return on investment in the economy exceeds the social discount rate; therefore when such returns are discounted by the social discount rate (allowing, where relevant, for the fact that part of the marginal returns may be saved), the present value of the returns will be greater than one. The result is that a unit of current savings has a greater value than a unit of current consumption for the value of a unit of savings is the discounted present value of future consumption it makes possible or $P^{INV} > 1$. For a more complete explanation see UNIDO, op. cit.

6 The UNIDO Guidelines uses aggregate consumption as its numeraire for measuring the benefits and costs of any project to the economy. To the extent that any resource input into a project is obtained at the expense of current consumption, the value of this input is correctly measured at market prices; to the extent, it is obtained at the expense of current savings it will be valued at the present value of future consumption that would have been available (i.e. at P^{INV}). For example, if the use of resources valued at V in a project reduces present savings and present consumption in the proportions X and 1-X respectively, the cost of these resources to the economy is $[(1-X) + X P^{INV}] V$. The O.E.C.D. Manual in contrast uses aggregate savings as its numeraire. To avoid confusion we will use the UNIDO methodology in the remainder of this paper. For those who prefer the O.E.C.D. method, we should point out that all the formulas for shadow wages in this paper using aggregate consumption as the numeraire may be converted into shadow wages using aggregate savings as the numeraire merely by dividing both sides of the equation by P^{INV}. For example, dividing both sides of (1) by P^{INV} will yield the familiar Little-Mirlees (O.E.C.D.) shadow wage rate on the basis of their additional assumption that $S^T = 1$.

expressed in market prices it accurately reflects the aggregate consumption gains to society because of the assumption that the marginal propensity to consume of workers (and their families) is unity. Note that if the value of P^{INV} is one the shadow wage rate is equal to Z (this would also be true in the somewhat less likely event that S^T is zero).

When the UNIDO argument is expressed in the above fashion, it is clear that the resulting formula suffers from two deficiencies. The first is the explicit assumption that workers (and their families) have a marginal propensity to save of zero. Thus, the formula is not theoretically general nor is it, we feel, a valid general proposition about the behavior of unskilled workers or rural households. Moreover, as we have shown elsewhere, this assumption is empirically false when applied to the case of low income urban and rural households in Thailand.[7] The second deficiency results from the implicit assumption that there are no direct costs to workers themselves or society in general from employment other than output foregone -- i.e. that there are no costs to the worker from, e.g., additional travel, foregone leisure time where longer working hours are involved, greater food consumption necessitated by greater work effort in project employment, psychic costs arising from differences in working conditions between the project and his alternative employment, etc. The UNIDO formula assumes that all these costs may be ignored and hence that any difference between the project wage W and output foregone Z may be regarded as a pure transfer payment from taxpayers to project workers. We see no reason why this assumption should be regarded as correct as a general proposition. Indeed, we feel that the correct expression for the direct costs to society from employing an additional worker is the marginal supply price of labor (call it L) -- i.e. in welfare economics terms, L is the minimum payment sufficient to induce the worker to accept employment on the project or, put differently, it is the output foregone in alternative employment plus that compensating variation sufficient to induce him to shift his employment to the project.[8] L may or may not be equal to the

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[7] See W. A. McCleary, Equipment Versus Employment - A Social Cost - Benefit Analysis of Alternative Techniques of Road Construction in Thailand (Geneva: International Labor Office; forthcoming).

[8] In emphasizing that it is the marginal supply price of labor and not merely output foregone which measures the direct costs of employing an additional worker on a project, we are following the lead of E. J. Mishan, Cost-Benefit Analysis (New York: Praeger Publishers, 1971), pp. 72-74 and A. C. Harberger, "On Measuring the Social Opportunity Cost of Labor" in A. C. Harberger, Project Evaluation-Collected Papers (Chicago: Markham Publishing Company, 1973), pp. 157-183.

project wage W; where W > L, it is W-L and not W-Z that measures the size of the transfer payment from taxpayers to project workers. In the remainder of this paper, we will derive a more general expression for the shadow wage rate which corrects the deficiencies in the UNIDO formulation and demonstrate that the expressions for the shadow wage commonly found in the literature on project evaluation and development economics are but special cases of our more general case.

First of all, let us generalize the UNIDO shadow wage rate to cover that case where the marginal propensity to save of workers (and their families) is no longer assumed to be zero. In this event, the shadow wage rate becomes:

$$
\begin{aligned}
(2) \qquad W^* &= Z[(1-S^W) + S^W P^{INV}] \\
&\quad + W[(1-S^T) + S^T P^{INV}] \\
&\quad - W[(1-S^W) + S^W P^{INV}] \\
&= Z[(1-S^W) + S^W P^{INV}] \\
&\quad + W[(S^T-S^W)(P^{INV}-1)]
\end{aligned}
$$

where S^W is the marginal propensity to save of worker households. We shall refer to (2) as the "Generalized UNIDO formula". It is easy to see that if $S^W = 0$ formula (2) collapses to formula (1). Moreover, if $P^{INV} = 1$, $W^* = Z$ regardless of the values of the marginal savings propensities of taxpayers and workers.

In taking account of the other costs of employment besides alternative output foregone, let us designate the amount $d = L-Z$ as those costs of employment aside from foregone output that must be borne by workers and their families and assume initially that the project wage rate is equal to the marginal supply price of labor -- i.e. $W = L$. By reasoning analogous to that used in the derivation of the preceding formulas, the new expression for the shadow wage rate becomes:

$$
\begin{aligned}
(3) \qquad W^* &= Z[(1-S^W) + S^W P^{INV}] \\
&\quad + d[(1-S^W) + S^W P^{INV}] \\
&\quad + W[(1-S^T) + S^T P^{INV}] \\
&\quad - W[(1-S^W) + S^W P^{INV}]
\end{aligned}
$$

which, since $Z + d = L = W$, becomes

$$W^* = W[(1-S^T) + S^T P^{INV}]$$

The value of the shadow wage no longer depends upon the marginal savings of workers or the value of these propensities relative to those of taxpayers. At first, this may appear to be a very strange result, but the explanation for it is straightforward: assuming that the project wage rate equals the marginal supply price of labor amounts to assuming that the welfare of the marginal worker is unaltered by employment on the project; the gain that workers obtain (W) just compensates them for the additional costs they incur (Z + d) and hence the gains and costs to workers from employment on the project, whether measured in terms of market prices or in terms of aggregate consumption costs, net out at zero. The only group that suffers a cost from employing an additional worker is taxpayers and it is their costs valued in terms of aggregate consumption that becomes the shadow wage rate.

The foregoing result may be made one step more general by dropping the assumption that the project wage is equal to the marginal supply price of labor. The elimination of this assumption allows us to derive an expression for the shadow wage rate which will cover those cases where unions or government minimum wage laws are successful in establishing a project wage that is above market clearing rates and those cases where the government for reasons of policy (e.g. income distribution) attempts to set project wages at a level different from the marginal supply price. In the event that $W \neq L$, the shadow wage rate becomes:

$$\begin{aligned} (4) \quad W^* &= Z[(1-S^W) + S^W P^{INV}] \\ &\quad + d[(1-S^W) + S^W P^{INV}] \\ &\quad - W[(1-S^W) + S^W P^{INV}] \\ &\quad + W[(1-S^T) + S^T P^{INV}] \\ &= L[(1-S^W) + S^W P^{INV}] \\ &\quad + W[(S^T-S^W)(P^{INV}-1)] \end{aligned}$$

which is perfectly analogous to formula (2) except that L has replaced Z in the formula. In contrast to formula (3), the values of L and S^W will influence the shadow wage rate because the discrepancy between

the project wage and the marginal supply price means that the gains and losses to workers from project employment will no longer net out at zero whether measured in terms of market prices or aggregate consumption costs. In the event that W = L, formula (4) obviously reduces to formula (3).

With the shadow wage rate expressed in formula (4), it is now possible to show that other expressions for the shadow wage commonly found in literature on development economics and on project evaluation in developing countries are all but special cases of our more general expression. In Table A we have listed some of the more common shadow wage rates, the authors usually identified with these expressions, and the (often implicit) assumptions about the parameter values in our formula (4) necessary to arrive at their shadow wage rate expressions. Note that in some cases several different sets of assumptions about parameter values may result in the same expression for the shadow wage. Note also that for formula (4) to be valid L must be positive but unequal to W, S^T and S^W must be both positive but unequal, and P^{INV} must be greater than 1; dropping one or more of these assumptions will result in different formulae for the SWR.

Before ending this paper, two additional points deserve emphasis. The first is that our formula (4) for the shadow wage rate is not just a theoretical nicety, of interest to economic theoreticians but of little or no practical value. Indeed, in our study referred to earlier, we used this very formula to derive a shadow wage for unskilled workers in the Northern and Northeastern regions of Thailand. This formula is of inherently more practical value than those versions of the SWR which involve alternative output foregone as one of the arguments. Estimates of output foregone involve rather intricate study of the consequences of withdrawing labor from agricultural production, studies which are difficult and time consuming to carry out and which of necessity are usually quite limited in application to fairly narrow locations and seasonal time periods. Of course, those versions of the SWR which state that it is best approximated at zero, or equal to the supply price of labor, or equal to the market wage rate appear much easier to apply than our formula but, as we have shown on the next page, their applicability is limited to those cases where the relevant set of parameter values in formula (4) can be expected to hold. On the other hand, our general formula stresses the relationship of the project wage rate and the supply price of unskilled labor. The latter can quite easily be estimated from wage surveys if such surveys classify labor by skill levels and by location. If such surveys do not exist, it is not unreasonable to expect that in many instances it would be practical

Table A Parameter Values Necessary for Formula (4) to Generate Common Expressions for the Shadow Wage Rate

Author	Shadow Wage Rate[1/]	Assumed Parameter Values
Lewis-Kahn	SWR = 0	Z=d=0 plus $P^{INV} = 1$ or $S^T = S^W$
———	SWR = Z	d=0 and $P^{INV} = 1$ or d=0 and $S^W = S^T = 0$
Harberger	SWR = L	$P^{INV} = 1$ or $S^T = S^W = 0$ or W=L and $S^T = 0$ (which yields one variant of the Galenson-Leibenstein SWR)
Galenson-Leibenstein	SWR = W	Z=d=0 plus a combination of parameter values for S^W, S^T, and P^{INV} such that $(S^T - S^W)(P^{INV}-1) = 1$ or W=L and $S^T = 0$, or W=L and $P^{INV}=1$
UNIDO	$SWR = Z+W\ s^T(P^{INV}-1)$	d=0 and $S^W = 0$
"Generalized UNIDO"	$SWR = Z\left[(1-S^W)+S^W P^{INV}\right] + W\left[(S^T - S^W)(P^{INV}-1)\right]$	d = 0
OECD[2/]	$SWR = Z+W(P^{INV}-1)$	$S^W = 0$, $S^T = 1$, d = 0

1/ The references for these expressions for the shadow wage rate are: W.A. Lewis, "Economic Development with Unlimited Supplies of Labour", Manchester School of Economic and Social Studies, Vol. 22 (1954), pp. 139-191; A. E. Kahn, "Investment Criteria in Development Programs, Quarterly Journal of Economics, Vol.LXV (1951), pp. 38-61; A.C. Harberger, "On Measuring the social Opportunity Cost of Labor" in A.C. Harberger, Project Evaluation - Collected Papers (Chicago: Markham Publishing Company, 1973), pp. 157-183; W. Galenson and H. Leibenstein, "Investment Criteria, Productivity, and Economic Development", Quarterly Journal of Economics, Vol. LXIX (1955), pp. 343-370; UNIDO, op.cit, pp. 201-212; and O.E.C.D., op.cit., pp. 157-177.

2/ Readers familiar with the Little and Mirrlees formula for the SWR expressed in their O.E.C.D. publication may not recognize this expression for their shadow wage. This confusion results from the fact that our formula for the SWR expresses costs in terms of aggregate consumption as the numeraire whereas Little and Mirrlees use aggregate savings as the numeraire. To obtain their result, it is merely necessary to divide both sides of the expression in the table by P^{INV} -- i.e. $\frac{SWR}{P^{INV}} = \frac{Z + W(P^{INV} - 1)}{P^{INV}} = W - \frac{1}{P^{INV}}\left[W - Z\right]$ which is the familiar Little and Mirrlees result in terms of aggregate savings as the numeraire.

for project evaluators to carry out their own surveys of wages for unskilled workers in the project area; this approach becomes difficult or impractical only where the project itself covers a large geographical area or where the project leads to such a massive increase in the demand for labor relative to local availability that a considerable migration of labor is necessary to fulfill the project demand (in this latter case, it is necessary to estimate at what wage will the project's demand for labor be met; such a wage will undoubtedly differ quite significantly from the going wage in the project locale). Admittedly there are problem in estimating the remaining parameters in our formula-the savings propensities of taxpayers and unskilled workers and the shadow price of savings, but it does not appear unreasonable to presume that plausible estimates can be made from national accounts data and household income expenditure surveys where such data exist. Nonetheless, it is important to stress the empirical limitations of any estimates of a shadow wage rate for unskilled labor. We have the impression that many people feel that given enough data it should be possible to estimate "the" SWR for Thailand. Before and after the start of our study, we have discovered through numerous casual interviews outside of Bangkok that there may be variations in daily wages for agricultural workers on the order of 25% for locales as close together as 20 kilometers and that these wages may vary by as much as 20% between wet season peaks and dry season lows. Such findings are not surprising for a country where communications and transportation systems are still primitive in many areas, knowledge of alternative opportunities is highly imperfect, and the degree to which rural households are integrated into the market system may vary considerably from place to place. Our interviews were hardly massive or systematic enough for us to attempt to incorporate them into our study, but they do suggest that one ought to be very cautious about the use of shadow wage rates. The shadow wage for unskilled labor in a particular region is at best an indication of rough order of magnitude and not a precise measure of the real costs to society of using unskilled labor regardless of project location within the region.

The second point worth emphasis is that the estimate of the SWR in our formula (4) - and indeed all the SWR's that are special cases of our general case -- ignore the income distribution goals of the government. Implicit in the formula is the assumption that society places the same value on a baht's worth of consumption whether by taxpayers or by unskilled workers and a baht's worth of saving regardless of which group does the saving (i.e. the consumption of either group is correctly measured by market prices and the saving of either group is valued by the shadow price of saving). This assumption is valid only if society feels that the existing income distribution is already optimal or that

redistributions of income will be handled solely by governmental tax and transfer policies. The former is unlikely and in the latter case important political and economic considerations usually constrain governments from achieving the optimal distribution. Governments are always constrained by what is acceptable to various power groups or to the population in general and there may be objections to particular kinds of taxes, progressive taxes may be difficult to administer at low levels of development, and pure subsidies to low-income groups seem to meet with criticism the world over. Moreover, the tax and transfer process may not be without real costs to the economy since both taxes and transfers affect the allocation of resources; in this event, it would appear foolish to pursue the government's distribution objectives by means of this tool alone. When such political and economic constraints are binding, both the aggregate consumption goal and the income distribution goal may enter the process of project selection. In this event, it is necessary to know what groups the government feels are worthy of special consideration[9] and what weights the government places on their consumption relative to the consumption of other groups. Then in the evaluation of any project, it is necessary to allocate all real benefits and costs and all transfer payments to the various groups and then revalue them in terms of the weights assigned to each group. Thus it is perfectly possible for a shadow wage rate to reflect both the aggregate consumption and income distribution objectives of the government. Our formula reflects only the former, but it would be feasible to derive an expression for the shadow wage rate by placing weights on the losses of taxpayers and the gains and losses of unskilled workers.

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[9] These groups may be identified by income but also by race, region, occupation, etc.; the income classification seems most desirable since there may be considerable income and hence welfare variation in groups classified by other criteria.