1 Model

The model is a new Keynesian DSGE model augmented by a financial sector, based on Bernanke et al. (1998)'s financial accelerator model. Two possible sources that lead to the asymmetric effects of the monetary policy are considered. First, the role of convex supply curve is captured by a nonlinear Phillips curve. Price stickiness is motivated by a Calvo (1983)'s pricing. Second, the role of financial friction is explained by a financial accelerator model, which is motivated by the asymmetric information between borrowers and a lender.

The model is composed of households, entrepreneurs, retailers, a bank and a government. The bank and entrepreneurs are introduced into the model to establish a financial accelerator. The rest of the model is a standard new Keynesian model. Households live infinitely. They supply their labor hours to entrepreneurs to receive wages. They obtain other sources of income from the interest payment from their deposit and the dividend from retailers. Households then allocate their incomes into consumption spending, money balance maintenance and interest-earning deposit holding.

Entrepreneurs compete in a perfect competition market. They produce intermediate goods by combining capital and labor service and then sell them to retailers. Such group plays an important role in a credit market. They are the borrowers in the model. To make entrepreneurs being borrowers, their life expectancy is assumed shorter than the households'. At each period, a fraction $1-\gamma$ of entrepreneurs will pass away and be born, making the number of entrepreneurs constant over time. Since their expected lifetime is shorter than that of the households', entrepreneurs are more impatient and inclined to borrow. This assumption also prevents entrepreneurs from having the sufficient funds enabling them to self-finance. Thus, in this model, entrepreneurs borrow funds from the bank to produce intermediate goods and sell them to retailers. The intermediate goods are sold in a monopolistic competition market.

Retailers produce differentiated products from intermediate goods and sell them to consumers in a monopolistic competition market. The bank lends money to the entrepreneurs whose investment projects are risky, and offers households a risk-free deposit. The government is responsible for both fiscal and monetary policies to stabilize the economy.

The key element of the model is the optimal contract between the entrepreneurs and the bank. There are two components of the investment risk in this model: an aggregate risk and an idiosyncratic risk. While the aggregate risk is public information, the idiosyncratic risk is entrepreneurs' private information. Since the optimal contract is essential to the model, it will be discussed first.

1.1 Optimal Contract

The bank and entrepreneurs are risk neutral. The bank raises funds by issuing a risk-free deposit to households. The entrepreneurs borrow funds from the bank to finance their investment projects. The value of investment project of each entrepreneur j is $Q_tK^j_{t+1}$, which is the nominal value of next period capital, K_{t+1}, at current period capital price, Q_t.

The entrepreneurs' gross return, $\omega^j_{t+1}R^k_{t+1}\left(Q_tK^j_{t+1}\right)$, is composed of the aggregate return on capital and the individual specific return or idiosyncratic risk. The aggregate return on capital, R^k_{t+1}, is public information and common to all entrepreneurs. The specific return, ω^j_{t+1}, is each entrepreneur's private information and i.i.d. across the entrepreneurs and periods with distribution $F(\omega^j_{t+1})$ and $E_t(\omega^j_{t+1}) = 1$ for all j.

The bank has to pay a fraction μ of the realized returns[1] to verify the actual returns and to liquidate the projects. Then the bank can keep the remainders, $(1-\mu)(\omega^j_{t+1}R^k_{t+1}Q_tK^j_{t+1})$. This costly state verification causes an agency cost problem. The entrepreneurs will have an incentive to misreport their realized returns. Thus, the optimal financial contract between the bank and the entrepreneurs is designed to make the entrepreneurs always report truthfully.

In this paper, the contract is assumed to be a one period contract. Even though a multi-period contract is more realistic, it is complicated to be derived in a context of general equilibrium and does not change the role of net worth on borrowing conditions (this assumption of the contract is similar to Bernanke and Gertler (1990) and Carlstrom and Fuerst (1997)).

The entrepreneurial sector is composed of a continuum of entrepreneurs in a unitary mass. At the end of period t, each entrepreneur j decides how much to invest by purchasing capital for the next period, K^j_{t+1} at price Q_t. Because the value of the investment exceeds the entrepreneur's net worth, N^j_{t+1}, the entrepreneur will have to make a loan, $B^j_{t+1} = Q_tK^j_{t+1} - N^j_{t+1}$, to finance the investment project. The bank and the entrepreneur will agree on a gross loan rate, Z_{t+1}.

The entrepreneur j will be able to pay back the loan if the project's return is greater than the obligation, i.e. $\omega^j_{t+1}R^k_{t+1}Q_tK^j_{t+1} \geqslant Z^j_{t+1}B^j_{t+1}$. Given a common aggregate return on capital, each entrepreneur's solvency condition will depend only on a specific return ω^j_{t+1}. The break-even specific return, $\overline{\omega}_{t+1}$, can be defined by:

$$\overline{\omega}^j_{t+1}R^k_{t+1}Q_tK^j_{t+1} = Z^j_{t+1}B^j_{t+1} \tag{1}$$

If the project is solvent, the entrepreneur pays back the obligation and keeps the surplus, otherwise the entrepreneur gets nothing. Assuming for a moment that the specific return is observable, the entrepreneur will not have any benefit from declaring bankruptcy if the project is solvent. Thus the entrepreneur's expected return is,

$$\int_{\overline{\omega}^j_{t+1}}^{\infty}(\omega^j_{t+1}R^k_{t+1}Q_tK^j_{t+1} - Z^j_{t+1}B_{t+1})dF(\omega^j_{t+1}).$$

Using definition of $\overline{\omega}^j_{t+1}$ in Equation (1) to eliminate Z^j_{t+1} leads to the entrepreneur's

[1] Carlstrom and Fuerst (1997) assume that the verifying cost is a proportion of investment value.

expected profit as a function of $\overline{\omega}^j_{t+1}$,

$$\int_{\overline{\omega}^j_{t+1}}^{\infty} (\omega^j_{t+1} R^k_{t+1} Q_t K^j_{t+1} - \overline{\omega}^j_{t+1} R^k_{t+1} Q_t K^j_{t+1}) dF(\omega^j_{t+1}),$$

$$R^k_{t+1} Q_t K^j_{t+1} \left\{ \int_{\overline{\omega}^j_{t+1}}^{\infty} \omega^j_{t+1} dF(\omega^j_{t+1}) - \int_{\overline{\omega}^j_{t+1}}^{\infty} \overline{\omega}^j_{t+1} dF(\omega^j_{t+1}) \right\},$$

$$R^k_{t+1} Q_t K^j_{t+1} f_{t+1}, \tag{2}$$

where $f_{t+1} = \int_{\overline{\omega}^j_{t+1}}^{\infty} \omega^j_{t+1} dF(\omega^j_{t+1}) - \int_{\overline{\omega}^j_{t+1}}^{\infty} \overline{\omega}^j_{t+1} dF(\omega^j_{t+1})$.

However, when the specific return is unobservable, the entrepreneur will have an incentive to declare bankruptcy if the profit obtained from the project is too low. To verify the entrepreneur's realized return, the bank has to pay a verifying cost proportional to the realized return, $\mu(\omega^j_{t+1} R^k_{t+1} Q_t K^j_{t+1})$. The net asset that the bank can obtain after verifying the realized return is $(1-\mu)(\omega^j_{t+1} R^k_{t+1} Q_t K^j_{t+1})$. Thus, the bank's expected income also depends on $\bar{\omega}$ and can be written as:

$$\int_{\overline{\omega}^j_{t+1}}^{\infty} Z^j_{t+1} B_{t+1} dF(\omega^j_{t+1}) + \int_0^{\overline{\omega}^j_{t+1}} (1-\mu)(\omega^j_{t+1} R^k_{t+1} Q_t K^j_{t+1}) dF(\omega^j_{t+1}),$$

$$R^k_{t+1} Q_t K^j_{t+1} \left\{ \int_{\overline{\omega}^j_{t+1}}^{\infty} \overline{\omega}^j_{t+1} dF(\omega^j_{t+1}) + \int_0^{\overline{\omega}^j_{t+1}} (1-\mu)(\omega^j_{t+1}) dF(\omega^j_{t+1}) \right\},$$

$$R^k_{t+1} Q_t K^j_{t+1} \left\{ (1 - F(\overline{\omega}^j_{t+1}))\overline{\omega}^j_{t+1} + \int_0^{\overline{\omega}^j_{t+1}} (1-\mu)(\omega^j_{t+1}) dF(\omega^j_{t+1}) \right\},$$

$$R^k_{t+1} Q_t K^j_{t+1} g_{t+1}, \tag{3}$$

where $g_{t+1} = (1 - F(\overline{\omega}^j_{t+1}))\overline{\omega}^j_{t+1} + \int_0^{\overline{\omega}^j_{t+1}} \omega^j_{t+1} dF(\omega^j_{t+1}) - \mu \int_0^{\overline{\omega}^j_{t+1}} \omega^j_{t+1} dF(\omega^j_{t+1})$.

Even though the specific return is unobservable to the bank, this idiosyncratic risk is perfectly averaged out by the large number of entrepreneurs. Thus, the only risk that the bank has to bear is the aggregate one. Assuming there are no other operating costs, the only cost of funds that the bank has is the opportunity cost of funds acquired from the deposit. Thus the bank's expected profit is reduced to a function of the aggregate capital, K_{t+1}

$$R^k_{t+1} Q_t K_{t+1} g_{t+1}. \tag{4}$$

Note that $f(\overline{\omega}^j_{t+1}) + g(\overline{\omega}^j_{t+1}) = 1 - \int_0^{\overline{\omega}^j_{t+1}} \mu(\omega^j_{t+1}) dF(\omega^j_{t+1})$. The second term appears because of the verifying cost μ. The total amount of the verifying cost depends on the level of $\overline{\omega}^j_{t+1}$, which depends on the loan rate, Z^j_{t+1}. The loan rate, Z^j_{t+1}, affects the bank's expected income in two opposite ways. If the entrepreneur can meet the loan obligation, a higher loan rate will increase the bank's income. However, a higher loan rate also increases the default risk. It will make the entrepreneur declares bankruptcy more often incurring a higher verifying cost to the bank.

The optimal contract is designed in the way that all entrepreneurs obtain a maximized profit and the bank still participates in the contact. If the expected discounted return to capital is defined as $S_t \equiv E_t[\frac{R^k_{t+1}}{R_t}]$, S_t can also be interpreted as the external finance premium (EFP). The optimal contract implies that:

$$S_t = \psi(\frac{Q_t K_{t+1}}{N_{t+1}}), \;\; with \;\; \psi(1) = 1, \;\; \psi'(\cdot) > 0. \tag{5}$$

Equation (5) is the key relationship of the model. The EFP is an increasing function of entrepreneur's leverage. Higher leverage level increases the default risk, escalating the cut off level, $\overline{\omega}^j_{t+1}$. Thus, the bank will charge a higher risk premium to compensate the risk.

1.2 The General Equilibrium

1.2.1 Entrepreneurs

The infinite numbers of entrepreneurs use identical constant return technology to produce wholesale goods and sell them to the retailers. Besides the investment decision described in the previous section, the entrepreneurs also supply labor service and consume consumption goods. The production function is a standard Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{6}$$
$$L_t = H_t^{\Omega} (H_t^e)^{1-\Omega}. \tag{7}$$

The aggregate wholesale goods, Y_t, is a function of aggregate capital, K_t, aggregate labor service, L_t, and a stochastic productivity, A_t. The aggregate labor service is a combination of the household labor supply, H_t and the aggregate entrepreneurs labor supply, H_t^e. By supplying labor service, entrepreneurs will have positive incomes even if the projects are default. To make the model comparable to a standard model, the labor income share of the entrepreneurial sector , $(1-\Omega)(1-\alpha)$, is assumed small. Furthermore, the entrepreneurial labor supply is assumed to be inelastic and normalized to one. Lastly, the productivity is an exogenous stochastic process following an AR(1) process,

$$ln(A_{t+1}) = \rho_a ln(A_t/\bar{A}) + \varepsilon^a_{t+1}, \tag{8}$$

where ε^a_t is an i.i.d. productivity shock and ρ^a is the AR coefficient.

The aggregate capital stock depends on the aggregate investment level, I_t and can be written as:

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) + (1-\delta)K_t, \tag{9}$$
$$\Phi\left(\frac{I_t}{K_t}\right) = \left(\frac{I_t}{K_t} - \frac{\Psi}{2}(\frac{I_t}{K_t} - \delta)^2\right) K_t, \tag{10}$$

where δ is the depreciation rate. A convex investment adjustment cost, $\Phi\left(\frac{I_t}{K_t}\right)$, is introduced to allow the asset price movement. As pointed out by Kiyotaki and Moore (1997), the variability of the capital contributes to the entrepreneurs' net worth volatility and enhances the fluctuation in business cycle. The adjustment cost is an increasing and concave function and equals to zero at the steady state. The capital price in terms of consumption good, Q_t, will be:

$$Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}. \tag{11}$$

The wholesale goods is sold to the retailers who will repackage and resell the products in a monopolistic competition at a higher price with a gross markup price X_t. Because the price of the consumption goods is normalized to one, the markup price is just the inverse of the nominal marginal cost, $X_t = \frac{1}{MC_t}$. The first order conditions and the Cobb-Douglas production function imply the following gross input returns:

$$R_t^k = \frac{\alpha \frac{Y_t}{K_t} MC_t + Q_t(1-\delta)}{Q_{t-1}}, \tag{12}$$

$$W_t = (1-\alpha)\Omega \frac{Y_t}{H_t} MC_t, \tag{13}$$

$$W_t^e = (1-\alpha)(1-\Omega)\frac{Y_t}{H_t^e} MC_t, \tag{14}$$

where R_t^k is the aggregate capital return and W_t and W_t^e are the wage payment for labor service from households and entrepreneurs, respectively.

Let V_t be the entrepreneur's realized return in period t. From Equation (2), the entrepreneurs' realized return is $V_t = R_t^k Q_{t-1} K_t^j f_t(\overline{\omega}_t^j)$. At the end of each period, there is a fraction, $1-\gamma$, of entrepreneurs who pass away and were born. The surviving entrepreneurs carry the project returns and their wages to the next period. The dying entrepreneurs spend all incomes on consumption. The dynamics of aggregate entrepreneurial net worth and aggregate entrepreneurial consumption are given by:

$$N_{t+1} = \gamma V_t + W_t^e, \tag{15}$$

$$C_t^e = (1-\gamma)V_t. \tag{16}$$

1.2.2 Households

Households live infinitely. In each period, they consume C_t, hold money balance $\frac{M_t}{P_t}$, and supply labor service H_t. Their incomes come from wage payment W_t, the benefits from deposits $R_t D_t$ and the dividends from the retailers F_t. Households also have to pay a lump-sum tax T_t every period. Households' optimization problem can be written as:

$$\max\ E_t\, \Sigma_{k=0}^{\infty} \beta^k U(C_{t+k}, \frac{M_{t+k}}{P_{t+k}}, H_{t+k}),$$

subject to

$$C_t = W_t H_t - T_t + F_t + R_t D_t - D_{t+1} + \frac{(M_{t-1} - M_t)}{P_t}.$$

The first order conditions implie that,

$$U_{C_t} = E_t(\beta U'_{C_{t+1}}) R_{t+1}, \tag{17}$$

$$-\frac{U_{H_t}}{U_{C_t}} = W_t, \tag{18}$$

$$\frac{U_{M_t/P_t}}{U_{C_t}} = \left(\frac{R^n_{t+1} - 1}{R^n_{t+1}}\right)^{-1}, \tag{19}$$

where

$$U(\cdot) = \frac{(C_t - \psi \bar{C})^{1-\sigma_C}}{1-\sigma_C} + \rho \frac{(1-H_t)^{1-\sigma_n}}{1-\sigma_n} + \xi \ln(\frac{M_t}{P_t}), \tag{20}$$

$$R^n_{t+1} = R_t \frac{P_{t+1}}{P_t}. \tag{21}$$

1.2.3 Retailers

The infinite number of retailers in a unit interval competes in a monopolistic competitive market. They purchase wholesale goods from entrepreneurs, repackage and resell them to the households at a higher price. The market friction makes room for monetary authority to influence the real economy. Let $Y(z)$ and $P(z)$ be the output and the price sold by the retailer z, respectively.

On the demand side, the final consumption goods, Y_t^f is a Dixit-Stiglitz composite of differentiated products. The final goods and its price index, P_t, are:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{(\varepsilon-1)/\varepsilon} dz\right]^{\varepsilon/(\varepsilon-1)}, \tag{22}$$

$$P_t = \left[\int_0^1 P_t(z)^{(1-\varepsilon)} dz\right]^{1/(1-\varepsilon)}, \tag{23}$$

where $\varepsilon > 1$ and $\frac{1}{\varepsilon}$ is the elasticity of substitution. At the equilibrium $\eta = \frac{\varepsilon}{\varepsilon - 1}$ defines the gross mark-up pricing. The demand curve facing each retailer is:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t^f. \tag{24}$$

Additionally, the price dynamics follows the Calvo type setting. At any period, a random fraction $1-\theta$ of the retailers have a chance to change their prices. The aggregate price index is a composite of a new optimal price, P_t^*, set by the fraction of retailers who can change their prices and the last period price, P_{t-1}, of which the fraction of retailers cannot change. The price index according to Calvo pricing evolves:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{25}$$

Retailer z who can change the price chooses P_t^* to maximize the expected real profit,

$$E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\frac{P_t^* Y_{t+k}(z)}{P_{t+k}} - \frac{TC(Y_{t+k}(z))}{P_{t+k}}\right], \tag{26}$$

where $\Lambda_{t,t+k} = \beta^k \frac{U'(C_{t+k})}{U'(C_t)}$ is the discount factor taken from the households sector and $TC(Y_{t+k}(z))$ is the total cost of producing $Y_{t+k}(z)$. Let MC_{t+k} be the marginal cost of producing an additional unit of $Y_{t+k}(z)$ and $\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t}$ be the cumulative inflation from period t to $t+k$. The first order condition implies:

$$\frac{P_t^*(z)}{P_t} = \frac{\eta E_t\left[\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} MC_{t+k} \Pi_{t,t+k}^{\varepsilon-1} Y_{t+k}^f\right]}{E_t\left[\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \Pi_{t,t+k}^{\varepsilon} Y_{t+k}^f\right]} = \frac{N_t}{D_t}. \tag{27}$$

Equation (25) and Equation (27) are the key equations of the Phillips curve. However, both equations are not particularly useful for computation purpose. Equation (25) can be rewritten as a function of inflation rate at period t, $\Pi_t = \frac{P_t}{P_{t-1}}$ by raising both sides by $1-\varepsilon$ and dividing by P_t,

$$\theta \Pi_t^{\varepsilon-1} = 1 - (1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon}.$$

The numerator N_t and de-numerator D_t in Equation (27) can be rewritten in a recursive structure which denotes each variable as a function of its next period variable.

$$\begin{aligned} N_t &= \eta E_t\left[\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} MC_{t+k} \Pi_{t,t+k}^{\varepsilon-1} Y_{t+k}^f\right], \\ &= \eta U_{C_t} MC_t Y_t^f + \theta\beta E_t\left[\Pi_{t+1}^{\varepsilon} N_{t+1}\right], \end{aligned}$$

and

$$\begin{aligned} D_t &= E_t\left[\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \Pi_{t,t+k}^{\varepsilon} Y_{t+k}^f\right], \\ &= U_{C_t} Y_t^f + \theta\beta E_t\left[\Pi_{t+1}^{\varepsilon-1} D_{t+1}\right]. \end{aligned}$$

The Calvo price dynamics in Equation (25) and the optimal pricing in Equation (27) are now rewritten as:

$$\theta \Pi_t^{\varepsilon-1} = 1 - (1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon}, \tag{28}$$

$$N_t = \eta U_{C_t} MC_t Y_t^f + \theta\beta E_t\left[\Pi_{t+1}^{\varepsilon} N_{t+1}\right], \tag{29}$$

$$D_t = U_{C_t} Y_t^f + \theta\beta E_t\left[\Pi_{t+1}^{\varepsilon-1} D_{t+1}\right]. \tag{30}$$

1.2.4 Government

The government conducts both fiscal and monetary policies to stabilize the economy. The government expenditure G_t is financed by money creation and lump-sum tax,

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t. \tag{31}$$

The government expenditure is governed by a stochastic process,

$$ln(G_{t+1}) = \rho_a ln(G_t/\bar{G}) + \varepsilon^g_{t+1}, \tag{32}$$

where g_t is a stochastic process, ε^t_t is the government expenditure shock and ρ_g is an AR coefficient. Monetary authority uses interest as a policy instrument. The nominal interest rate R^n_t follows Taylor rule. It is set to respond to the deviation of output and inflation.

$$R^n_t = \overline{R}\left(\frac{R^n_{t-1}}{\overline{R}}\right)^{\rho_r}\left(\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\rho_\pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\rho_y}\right)^{1-\rho_r} mp_t, \tag{33}$$

where the variables with a bar are steady state values and , $\rho_r > 0$, $\rho_{pi} > 0$ and $\rho_y > 0$ are the policy coefficients of the policy rule. The monetary policy shock is governed by an exogenous process:

$$ln(mp_{t+1}) = \rho_{mp} ln(\frac{mp_t}{\bar{mp}}) + \varepsilon^{mp}_{t+1}. \tag{34}$$

Since the nominal interest rate is the policy instrument, the money stock M_t in Equation (31) is determined endogenously by money demand in Equation (19). Thus, the lump-sum tax T_t in Equation (31) is endogenously adjusted to balance the government budget.

1.2.5 Equilibrium Conditions

Final goods are distributed to private consumption, investment and government consumption. The aggregate verifying cost is represented in the last term in the following equation:

$$Y^f_t = C_t + C^e_t + G_t + I_t + \int_0^{\overline{\omega}^j_t} \mu(\omega^j_t) dF(\omega^j_t). \tag{35}$$

Intermediate goods market clearing condition:

$$Y_t = \int_0^1 Y_t(z)dz = Y^f_t \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz = Y^f_t \Delta_t, \tag{36}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$ is the price dispersion. Using Calvo pricing dynamics, the dynamics of the price dispersion is,

$$\Delta_t = \theta \Delta_{t-1} \Pi^\varepsilon_t + (1-\theta)\left(\frac{N_t}{D_t}\right)^{-\varepsilon}. \tag{37}$$

Financial market clears the aggregate borrowing from entrepreneurs and deposit from the households:

$$B_t = D_t. \tag{38}$$

References

Bernanke, B. and M. Gertler (1990). Financial fragility and economic performance. *The Quarterly Journal of Economics 105*(1), 87–114.

Bernanke, B., M. Gertler, and S. Gilchrist (1998, March). The financial accelerator in a quantitative business cycle framework. NBER Working Papers 6455, National Bureau of Economic Research, Inc.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics 12*, 383–398.

Carlstrom, C. and T. Fuerst (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *The American Economic Review 87*(5), 893–910.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of political economy 105*(2), 211–248.