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Portfolio Optimisation under Copula-Based Scenarios

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Abstract

Optimising a multicurrency portfolio requires a flexible model to manage exchange rate risk as well as representational data on asset-currency dependency. Additionally, deliberate scenario generation is also vital for portfolio risk evaluation, especially for the case of extreme events. This study proposes a mean-CVaR portfolio optimisation model with currency overlay under regular-vine copula generated scenarios. To highlight the importance of scenario generation technique, performances of the resulting portfolios from the proposed method are compared with those optimised under multivariate normal assumption. The results show that portfolios from our proposed approach outperform those from traditional method, both in return and risk metrics. The outperformance is largely attributed to active currency hedging, which takes advantage of detailed information captured by a regular-vine copula.

Keywords: international portfolio, currency overlay, regular-vine copula

JEL Codes: C15, C61, G11

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1 Introduction

The seminal work of Markowitz [1] formulated the portfolio optimisation problem by minimising risk, the variance of the returns, at a given level of mean or expected return. Later versions of portfolio optimisation problems replace variance with Conditional Value-at-Risk (CVaR) to allow better capture downside risk. An implementation of mean-CVaR portfolio is introduced in Rockafellar and Uryasev [2] which becomes a standard practice for practitioners. However, the quality of CVaR risk measure depends largely on modelling portfolio return distribution.

Typical methods characterise probability distributions by the first to the fourth statistical moments; that are, mean, variance (or standard deviation), skewness and kurtosis. The relationship between different (marginal) distributions is generally described by a correlation matrix (see Høyland et al. [3] and Kaut et al. [4] for examples). Although these are frequently sufficient to capture the shape of marginal distributions, such statistical moments and correlations have limitations in describing the relationship between distributions. Correlation, in particular (specifically the correlation coefficient, or Pearson's correlation [5]), captures only linear dependency between pairwise variables, which can be misleading when the measured data contains outliers or is highly skewed. In such cases, a rank correlation, such as Kendall's tau [6], can be used to comprehend a non-linear relationship. Nonetheless, financial modelling often assume that asset returns are normally distributed and describe asset relationships with correlation.

Most financial security returns, in reality, are non-Gaussian and exhibit asymmetric dependence (see, for instance, Erb et al. [7], Longin and Solnik [8], Ang and Bekaert [9], Ang and Chen [10], Campbell et al. [11], Mitchell and Pulvino [12] and Patton [13]) In which returns are more strongly correlated in bear markets than in flat or bull markets. As a result, the return distribution generated under normality and linear dependence assumptions does not reflect realistic events, which has the potential to significantly affect the quality of solutions obtained by optimisation under such assumptions. We hence adopt copulas to model the dependence structure of asset returns.

Sklar [14] defines a copula as a function that connects the margins of a multidimensional distribution. The mathematical formulations of copulas are given in Sklar [15] and Nelsen [16]. Copulas have the major advantage of separating the marginal distributions and their dependency structures, allowing these components to be modelled independently. Unlike a standard multivariate normal distribution, which assumes all marginal distributions are Gaussian and have linear dependence, this feature allows you to combine marginal distributions from different families within copulas. Copulas allow the resulting return distribution to account for non-normality such as heavy tails and asymmetric dependencies. Such advancements have the potential to aid in the avoidance of risk underestimation in standard methods such as generating a distribution from a multivariate normal distribution.

One of this paper's contributions is an improvement in modeling a joint return distribution. In the existing literature, [17] and [18] use empirical copulas whilst Sutiene and Pranevicius [19] employ Gaussian and Student's t multivariate copulas in building return distribution. The drawback of such empirical copulas is that the solutions could be unreliable when derived from small samples. Furthermore, using multivariate copulas necessitates a dependence structure that can be described by a single copula family, which lacks flexibility when applied to high-dimensional modelling. To address these issues, we use vine copulas [20] to model the dependence structure. A vine copula is essentially a collection of bivariate copulas that can be represented by a nested set of trees that satisfy a specific set of conditions [20]. Vine copulas benefit from the ability to employ a diverse set of bivariate copula families. A dependence structure of random variables will then be estimated pair by pair, a more flexible process than being defined by a single copula family. The resulting return distribution generated by vine copulas should thus have the potential to better represent financial return dependencies.

The vine-copula model is then applied to real-world data to formulate a mean-CVaR international portfolio optimisation problem. Another contribution of this paper is the formulation of an optimisation model. The formula also includes a currency overlay built with foreign exchange forwards to allow for portfolio currency exposure adjustments. The optimisation model incorporates constraints related to currency overlay and portfolio transactions. Costs associated with exchange rate hedging that affect portfolio risk and return are also considered.

The effects of a new return distribution construction method and related constraints are empirically investigated in order to determine whether the new approach produces portfolios that are more resilient to extreme events than the standard approach. Portfolios are optimized with two types of distributions in our two-stage stochastic optimisation problem. One from the Regular-Vine-Copula-based approach, also known as a “RVC portfolio”. Another method is based on the assumption that asset returns are normally distributed and that the relationship between asset returns is described by correlation. Generally, a sampling is performed on a Multivariate Normal distribution; portfolios optimized with these return distributions are referred to as an “MVN portfolio”. Experiments are carried out to assess portfolio performance under the two types of return distributions.

The rest of this paper is organised as follows: Section 2 demonstrates an approach using a regular-vine copula to generate scenarios, a construction of currency overlay and a formulation of the optimisation model. Section 3 exhibits experiment results with analyses and section 4 provides a conclusion of this study

2 Methodology

2.1 Scenario Generation with Regular-Vine Copula Dependence Structure

In this paper, we describe a procedure for generating scenarios for our optimisation problem. To summarise, we use empirical marginal distribution to avoid making assumptions about asset return distribution. Then, to account for nonlinear dependency, construct a joint distribution using a regular vine copula.

1. *Modelling marginal distributions* - We fit an invertible empirical distribution to each time series of financial returns and estimate a marginal probability distribution function given empirical data in the form of time series of financial returns (PDF). To estimate an empirical PDF of a return time series, we use kernel density estimation (KDE). To approximate a probability density function f of a random variable Ξ_i for $i = 1, \ldots, n$ assuming that we have m independent observations $\xi_{i1}, \ldots, \xi_{im}$ on each random variable, a kernel density estimator for the estimation of the density value at point x is defined as

$$f(\xi_i) = \frac{1}{mh} \sum_{j=1}^{m} K\left(\frac{\xi_{ij} - \xi_i}{h}\right) \tag{1}$$

where K denotes a so-called kernel function, and h is a bandwidth. In our study, we select the Epanechnikov kernel as a kernel function and choose an optimal bandwidth according to Silverman's rule of thumb, [21].

An empirical cumulative distribution function (CDF) of each return series can be subsequently produced from an estimated PDF as follows:

$$F(\xi_i) = \sum_{\xi_{ij} \leq \xi_i} f(\xi_{ij}). \tag{2}$$

The resulting CDF is uniform on the interval $[0, 1]$ and is an input argument to a copula function. In what follows, we denote a CDF of a random variable i by u_i.

2. *Estimating a regular-vine copula* - To fit an R-Vine copula to a given dataset, Dissmann et al. [22] outline the procedure as follows:

   (a) Selection of the R-Vine structure, i.e. selecting which unconditioned and conditioned pairs to use for the pair-copula construction.

   (b) Fitting a pair-copula family to each pair selected in (a).

(c) Estimation of the corresponding parameters for each copula.

The sequential method developed by Dissmann [23], which fits an R-Vine tree-by-tree, is used in our study to estimate the R-Vine copula. In our study, an R-Vine copula model is estimated using the `VineCopula` package on R, [24]. The results of this step provide the most appropriate combination of bivariate copulas and conditional bivariate copulas (with respect to an R-Vine structure) for the given data.

3. *Sampling from a regular-vine density* - We follow the R-Vine sampling method from [22]. This step starts with sampling $u_1, \ldots, u_n$ which are independent and uniform on $[0, 1]$ then set

$$\begin{aligned} \xi_1 &= u_1, \\ \xi_2 &= F^{-1}_{2|1}(u_2|\xi_1), \\ \xi_3 &= F^{-1}_{3|12}(u_3|\xi_1, \xi_2), \\ &\vdots \\ \xi_n &= F^{-1}_{n|12\ldots n-1}(u_n|\xi_1, \ldots, \xi_{n-1}) \end{aligned} \tag{3}$$

where $F^{-1}_{j|12\ldots j-1}(u_n|\xi_1, \ldots, \xi_{j-1})$ for $j = 1, \ldots, n$ is the inverse of conditional cumulative distribution function defined by Joe [25]. A set of equations (3) yields dependent $\xi_1, \ldots, \xi_n$ which are equivalent to a collection of single scenarios of n random variables. To generate N scenarios, the random sampling of $u_1, \ldots, u_n$ is repeated N times.

2.2 Formulating a Conditional Value-at-Risk International Portfolio Optimisation Problem

In addition to market risk of asset returns, an international portfolio includes exchange rate risk. One method for managing exchange rate risk is to use currency overlay, which are adjustments made to currency exposure to speculate or hedge exchange rate risk according to the investor's preferences using exchange rate derivatives. The following optimisation problem formulation is adapted from the work of Chatsanga and Parkes [26].

2.2.1 Structure of A Portfolio with Overlay Constraints

	Hedged			Unhedged		
	US	UK	JP	US	UK	JP
asset exposure (%)	35	45	20	35	45	20
currency exposure (%)	27	55	18	35	45	20
overlay position (%)	-8	10	-2		-	
total overlay (%)		10			-	

Table 1: Sample portfolios with and without currency overlay.

When constructing a portfolio that invests in multiple countries, two types of returns influence the portfolio's market value. The first comes from asset prices plus dividends or other interest-bearing income, while the second comes from currency gains or losses. As a result, each country's investment is represented as a mix of exposure to asset markets and exposure to currency rates. This structure also facilitates changing currency exposure and thus dispersing risky foreign currency positions. Currency overlay is defined as a change in currency exposure that affects an unhedged portfolio's initial currency holdings.

A currency overlay is made up of overlay positions as depicted in Table 1. The overlay positions result from holding one or more foreign exchange forward contracts (FX forwards) as exemplified in Table 2. Each forward contract entails a "cost of carry" or hedging cost, which can be positive or negative depending on the interest rate differential between the currency pairs. Consider a portfolio that includes three foreign currency forwards as given in Table 2. Each forward contract's cost of carry is determined by the currency sold or purchased, the corresponding interest rate, and the position held in the portfolio. Selling JPY for USD at 1% of the portfolio, for example, results in a positive carry of $1\% \times 2\% - 1\% \times 1\% = 0.01\%$ to the portfolio. However, selling GBP for JPY results in a negative carry of $-2\% \times 4\% + 2\% \times 1\% = -0.06\%$ as a result of transferring exposure from a country with a high interest rate to a country with a low interest rate. The total overlay position accounts for 8% of the portfolio, with a positive carry of 0.13% from the three forward contracts combined. This amount of carry is added to the overall return of the portfolio.

	USD	GBP	JPY	Cost of Carry
interest rate (%)	2	4	1	
sell JPY, buy USD (%)	**1**		**-1**	0.01
sell USD, buy GBP (%)	**-9**	**9**		0.18
sell GBP, buy JPY (%)		**-2**	**2**	-0.06
overlay (%)	**-8**	**7**	**1**	0.13

Table 2: Costs of carry associated with forward contracts on foreign exchange rates. The numbers in bold indicate portfolio positions (in percentage). The total currency overlay position on each currency is calculated using the respective currency's net forward positions. The weighted sum of interest rates and forward positions with respect to the currencies associated with the forward contract is the cost of carry from holding each forward contract.

Therefore, the net cost of carry is the sum of interest rates and overlay positions. For an investment in any country j, the total return contributes to the portfolio is

$$r_j = a_j r_j^a + c_j r_j^c + v_j i_j \tag{4}$$

where r_j is total return from investment in country j; a_j, c_j and v_j are respectively asset exposure, currency exposure and overlay position on country j; r_j^a, r_j^c and i_j are respectively expected asset return, expected currency return and expected interest rate of country j.

Since an overlay position is defined as the difference in currency and asset exposures, equation (4) can be equivalently expressed as

$$\begin{aligned} r_j &= a_j r_j^a + c_j r_j^c + (c_j - a_j) i_j \\ &= a_j (r_j^a - i_j) + c_j (r_j^c + i_j). \end{aligned} \tag{5}$$

We define $r_j^a - i_j$ and $r_j^c + i_j$ as the adjusted return of asset and currency, respectively. Equation (5) demonstrates that the portfolio total return (return from assets, currencies, and foreign exchange forward carry costs) equals the product of adjusted returns, asset exposure, and currency exposure. This means that the formulation of overlay positions is not required for calculating a portfolio's total return. Furthermore, if a portfolio has no forward contracts, the interest rate terms in equation (5) are cancelled out, demonstrating that the formulation in equation (5) generalises the total return calculation of international portfolios.

To create a portfolio with currency overlay, denote $\mathbf{f}_k = (f_{k1}, ..., f_{kC})$ a vector of exposure from a forward contract k where $K = \binom{C}{2}$ is the total number of forward contracts available for investing in C countries. The forward contract specification implies that only two elements of $\mathbf{f}_k$ represent the exposure, one of which is equal to a negative value of another, while the rest of the elements only take a value of zero. To avoid putting those requirements into the constraints of an optimisation problem, we define f_{kj} as an element of a matrix $\mathbf{F}$ in which

$$\mathbf{F} \stackrel{\text{def}}{=} \mathbf{T} \circ (\mathbf{1}^{\mathrm{T}} \otimes \mathbf{q}) \tag{6}$$

where $\circ$ is the Hadamard product operator, $\otimes$ is the Kronecker product operator, $\mathbf{T}$ is a $K \times C$ combinatorial matrix of $\{-1, 0, 1\}$, $\mathbf{1}$ is a $C \times 1$ column vector of ones and $\mathbf{q}$ is a $K \times 1$ column vector determining the size of exposure. Thorough details on the formulation described earlier can be found in [26].

	Country 1	$\cdots$	Country j	$\cdots$	Country C
Asset class 1	a_{11}	$\cdots$	a_{1j}	$\cdots$	a_{1C}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
Asset class i	a_{i1}	$\cdots$	a_{ij}	$\cdots$	a_{iC}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
Asset class A	a_{A1}	$\cdots$	a_{Aj}	$\cdots$	a_{AC}
Forward position 1	f_{11}	$\cdots$	f_{1j}	$\cdots$	f_{1C}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
Forward position k	f_{k1}	$\cdots$	f_{kj}	$\cdots$	f_{kC}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
Forward position K	f_{K1}	$\cdots$	f_{Kj}	$\cdots$	f_{KC}
Asset exposure	$\sum_{i=1}^{A} a_{i1}$	$\cdots$	$\sum_{i=1}^{A} a_{ij}$	$\cdots$	$\sum_{i=1}^{A} a_{iC}$
Overlay position	$\sum_{k=1}^{K} f_{k1}$	$\cdots$	$\sum_{k=1}^{K} f_{kj}$	$\cdots$	$\sum_{k=1}^{K} f_{kC}$
Currency exposure	$\sum_{i=1}^{A} a_{i1} + \sum_{k=1}^{K} f_{k1}$	$\cdots$	$\sum_{i=1}^{A} a_{ij} + \sum_{k=1}^{K} f_{kj}$	$\cdots$	$\sum_{i=1}^{A} a_{iC} + \sum_{k=1}^{K} f_{kC}$
Total overlay	$\frac{1}{2}\sum_{j=1}^{C}\left\|\sum_{k=1}^{K} f_{kj}\right\|$				

Table 3: Structure of an international portfolio with currency overlay.

2.2.2 The Optimisation Problem

Based on the currency-overlaid portfolio structure outlined in Table 3, the portfolio optimisation problem can be set up with the following notations,

a Vector of asset exposure; $\mathbf{a} = [a_{11}, ..., a_{ij}, ..., a_{AC}]^{\mathrm{T}}$.

c Vector of currency exposure; $\mathbf{c} = [c_1, ..., c_j, ..., c_C]^{\mathrm{T}}$ where $c_j = \sum_{i=1}^{A} a_{ij} + \sum_{k=1}^{K} f_{kj};\ j = 1, ..., C.$

$\mathbf{x}$ Vector of decision variables; $\mathbf{x} = [\mathbf{a}, \mathbf{c}]^{\mathrm{T}}$.

$\mathbf{r}$ Vector of adjusted returns; $\mathbf{r} \in \mathcal{R}^{C(A+1)}$.

μ Target return of a portfolio.

The vector $\mathbf{r}$ contains adjusted expected returns of assets and currencies according to equation (5) by deducting expected interest rates from asset returns and adding them to currency returns. The conditional value-at-risk (CVaR) portfolio risk measure is used to calculate the true downside risk of the joint distribution modelled in Section 2.2.

The mean-CVaR portfolio optimisation problem with overlay constraints is subsequently formulated as:

$$\text{minimise:} \quad \alpha + \frac{1}{h(1-\beta)} \sum_{d=1}^{h} u_d \tag{7a}$$

$$\text{subject to:} \quad \mathbf{x}^{\mathrm{T}}\mathbf{r}_d + \alpha + u_d \geq 0, \tag{7b}$$

$$\mathbf{x} = [\mathbf{a}, \mathbf{c}]^{\mathrm{T}}, \tag{7c}$$

$$\mathbf{x}^{\mathrm{T}}\mathbf{r} = \mu, \tag{7d}$$

$$\mathbf{F} = \mathbf{T} \circ (\mathbf{1}^{\mathrm{T}} \otimes \mathbf{q}), \tag{7e}$$

$$c_j = \sum_{i=1}^{A} a_{ij} + \sum_{k=1}^{K} f_{kj}; f_{kj} = \mathbf{F}_{kj}, \tag{7f}$$

$$\frac{1}{2} \sum_{j=1}^{C} \left| \sum_{k=1}^{K} f_{kj} \right| \leq V_u, \tag{7g}$$

$$\mathbf{1}^{\mathrm{T}}\mathbf{a} = 1, \tag{7h}$$

$$\mathbf{1}^{\mathrm{T}}\mathbf{c} = 1, \tag{7i}$$

$$u_d \geq 0, \tag{7j}$$

$$0 \leq a_{ij} \leq 1. \tag{7k}$$

The mathematical expression of CVaR in an optimisation problem is referred to what introduced in the study of Rockafellar and Uryasev [2]. In our proposed optimisation problem, the linear expression of CVaR objective is expressed in (7a) and constrained by auxiliary variables u_d in (7b). It should be noted that VaR is calculated on the basis of an approximation of a continuous joint distribution of asset returns. To simplify the VaR function, the true distribution is discretised into d bins. Constraint 7d sets the portfolio target return. Constraints 7e and 7f formulate overlay positions. Constraint 7g limits the total overlay position to avoid excessive currency risk.

3 Results and Discussions

The preceding section demonstrates how to construct an international portfolio with a CVaR objective. This section describes how the portfolio was implemented using real-world data. The modelling of return distribution (whether or not nonlinear dependence of asset returns is taken into account) for CVaR calculation is the key driver for the resulting portfolios. The following experiment shows how it affects portfolio allocation and performance.

3.1 Data

In our study, the investments of interest are blue-chip stock indices, government bond indices and currencies. A portfolio invests in five major countries, i.e., the United States (US), the United Kingdom (UK), the Eurozone (EU), Japan (JP) and China (CN). The base currency of the portfolio is USD.

The data is collected on a monthly basis. The in-sample period runs from January 2004 to December 2018, and the out-of-sample period runs from January 2019 to September 2022. J.P.Morgan Markets collects government and corporate bond returns. Bloomberg provides currency and stock indices returns.

3.2 Scenario Generation Results

In our research, we create scenarios using two methods: RVC and MVN. The first uses an R-Vine copula to model asset dependence structure on a return distribution with no parametric form (RVC). The latter assumes that asset returns follow a multivariate normal distribution (MVN), with correlations describing the dependence structure. The assumptions used in the two methods result in different scenarios.

We follow the Monte Carlo simulation techniques of Levy [27] in order to construct scenarios based on a multivariate normal distribution (MVN). For generating RVC scenarios, the methodology is as described in Section 2.2. We allow for 5 bivariate copula families, namely Gaussian, Student's t, Clayton, Gumbel and Frank and the rotated version of Clayton and Gumbel copulas (at 90/180/270 degrees) in order to capture broader range of asset dependence structure. [28] and [16] provide extensive details on the bivariate copula families.

The example of generated scenarios from RVC and MVN approaches are illustrated in Figure 1. Since MVN assumes normality, the shapes of return distributions are symmetric. This implies that the downside and upside have equal frequencies. RVC-generated samples, on the other hand, have asymmetric distributions that preserve unbiased information from the raw data. As a result, the RVC distributions have more outliers and an imbalance of downside and upside events. When compared to other asset classes, the different characteristics of scenarios generated by the two methods are the same, and they are the key drivers influencing portfolio allocations in subsequent studies.

3.3 Experimental Studies

3.3.1 Efficient portfolios from Different Scenario Generation Methods

Our study highlights the effects of different assumptions made on scenario generation. The first approach generates scenarios by assuming that return distributions of securities are normally distributed and the co-movement between assets is represented by correlations

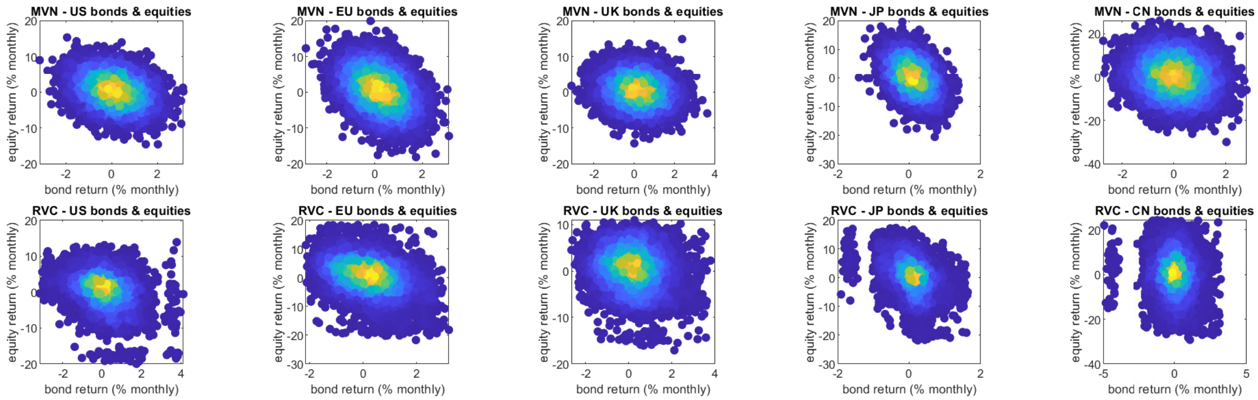

Figure 1: A comparison of bond and equity simulated returns generated with multivariate normal distribution (MVN) and R-Vine copula (RVC) for all countries. It is noticed that the RVC joint return distributions contain more extreme observations and their dependencies are asymmetric.

(or linear relationships). Another approach makes no assumption on distribution family, hence employing empirical distributions from available historical data. An inter-relationship between assets is captured by copulas. Thus, the solutions derived from the two types of generated scenarios differ in terms of the assumption on the shape of return distributions and the linear or non-linear relationship between securities.

Naturally, optimal portfolios are most efficient when evaluated in the "environment" they were created. The environment is defined as the returns generated under various underlying assumptions. For example, when the return distribution of one asset is assumed to be skew and fat-tailed rather than Gaussian, the return and risk of that asset can be completely different. Thus, optimal portfolios based on the multivariate normal return distribution (MVN) may not be the most efficient when their risks and returns are compared to return scenarios generated by other methods, such as the regular-vine copula based scenario (RVC). The efficient frontiers in Figure 2 are generated by applying optimal allocations to returns generated by scenario generators. Naturally, optimal portfolios in one environment are inefficient when tested in another. Different assumptions about return distributions have a significant impact on portfolio allocations and performance.

Consequently, we compare optimal allocations generated by two scenario generation

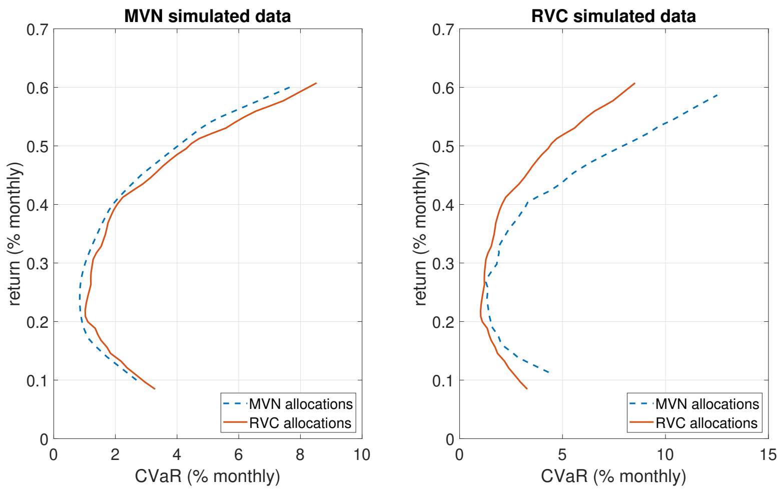

Figure 2: Comparison of efficient portfolios under different environments. The RVC portfolios are optimal under scenarios that do not assume normal distribution of returns. The MVN portfolios, on the other hand, are optimal under normal-distribution assumption. The left panel depicts the risk-return of efficient portfolios if returns are normally distributed and the right panel illustrates the risk-return profile of same portfolios if return distributions are not assumed Gaussian. Naturally, optimal portfolios in one environment are less efficient when evaluated in other environments.

methods to see if different assumptions cause deviations in optimal allocations. The left panel of Figure 3 illustrates equity allocations and the right panel shows foreign currency (non-USD) exposure of portfolios. High equity holdings and foreign currency exposure, in general, constitute risky portfolios. The optimal portfolios from the two methods have roughly the same equity proportions. This implies that deviations from the normality assumption have little impact on bond-equity allocation.

When it comes to hedging exchange rate risk, the MVN and RVC portfolios differ significantly from similar equity holdings. The RVC portfolios never hedge currency risk for less than 25%, whereas the MVN hedges for less than 20%, especially when risk appetite is high. This portrays various perspectives on risk and interdependence under various assumptions. Because the normality assumption cannot fully capture tail risk and tail dependence, risk from extreme events (particularly foreign exchange rate movements) may be underestimated.

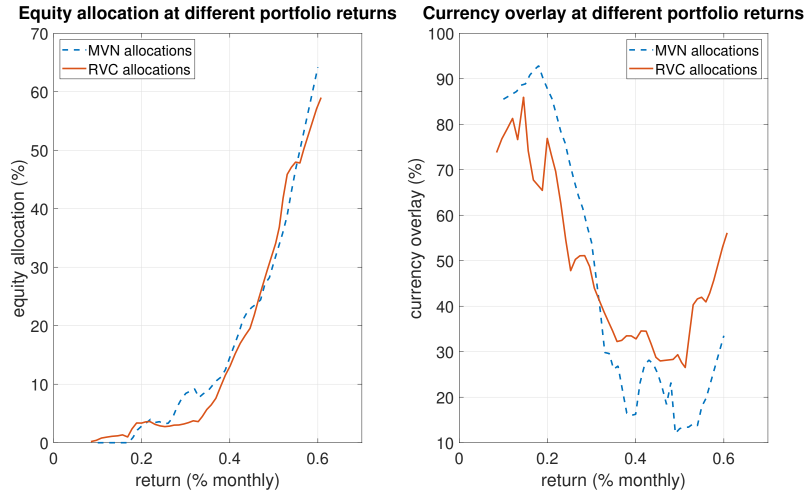

Figure 3: Equity allocation and foreign currency exposure of portfolios. The left panel shows proportion of equity (from all markets) in portfolios and the right panel presents total currency hedging position (currency overlay).

3.3.2 Portfolio Performances

Over the out-of-sample period, this study presents the cumulative returns of optimal portfolios generated by the two scenario generation methods (January 2019 to September 2022). The cumulative return index is shown in Figure 4 to compare compounded returns with an initial wealth of $100 in December 2018. To reflect different risk appetites, we choose three distinct optimal portfolios from the efficient frontiers based on target returns: high, medium, and low.

Table 4 provides descriptive statistics to cumulative returns in Figure 4. It is worth noting that RVC portfolios dominate MVN portfolios in all metrics but minimum return. The outperformance of RVC portfolios is also consistent across risk appetite levels.

Statistic	MVN			RVC		
	low	medium	high	low	medium	high
mean (%)	-0.04	0.13	0.64	-0.12	0.02	0.60
std dev (%)	0.67	0.73	2.41	0.87	0.81	2.80
min (%)	-1.28	-2.31	-6.82	-1.96	-2.39	-8.40
max (%)	1.54	1.61	5.60	1.54	2.14	7.23
max drawdown (%)	-6.10	-5.34	-10.18	-9.32	-7.13	-12.51

Table 4: Descriptive statistics on out-of-sample performances of MVN and RVC portfolios. RVC portfolios have better risk and return characteristics across risk appetite levels.

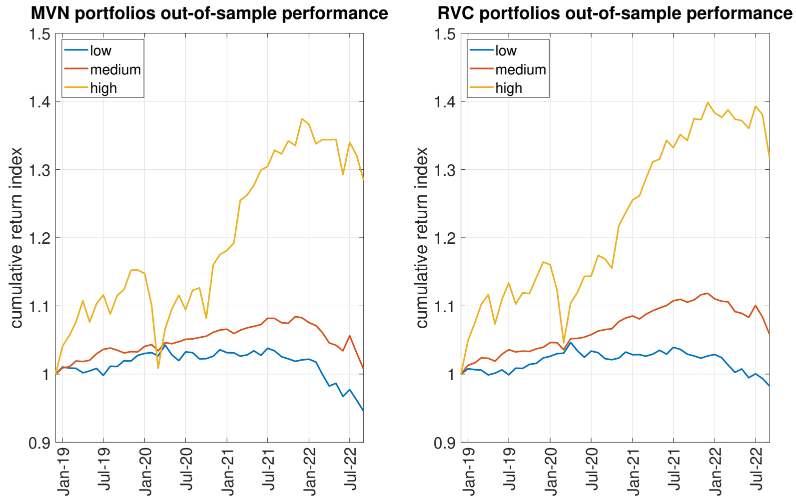

Figure 4: Cumulative wealth over the out-of-sample period (January 2019 to September 2022) of optimal portfolios from two scenario generation methods; RVC and MVN approaches. The curves represent cumulative wealth of optimal portfolios with high, medium and low target returns.

One possible explanation for RVC portfolio construction's superior performance is different currency hedging perspectives. Figure 5 illustrates the mean returns and CVaRs of assets as well as the FX return in USD (all adjusted for the cost of carry as shown in Section 2.2.1).

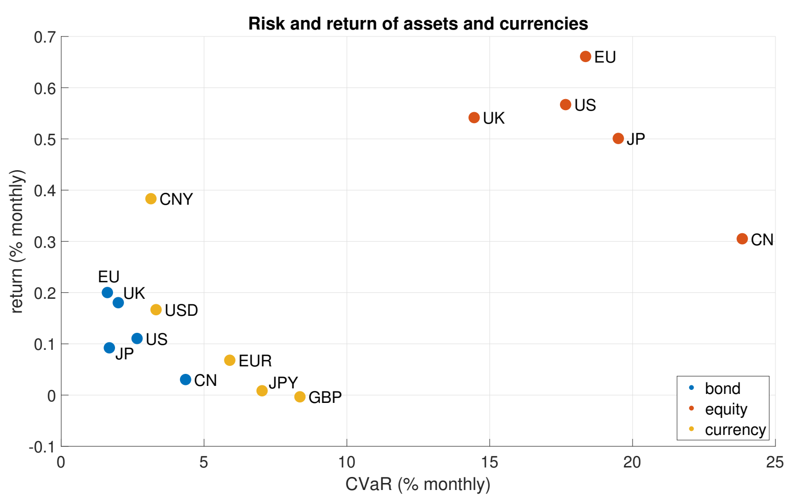

Figure 5: Risk (CVaR) and return (historical average of in-sample data) profile of assets and currencies (FX return in USD) for portfolio construction. The risk and returns calculated are cost of carry adjusted.

It can be seen from the figure that US, EU and UK equities provide higher returns with lower risk than the others. When a portfolio requires higher risk appetite, equity allocation must increase, particularly for US, EU and UK. Nevertheless, increasing equities from these countries also bear FX risk which EUR and GBP are among the highest. The best strategy is to secure equity return with minimum FX risk, which can be accomplished by currency overlay. Because RVC portfolios engage in more FX hedging (see Figure 3), superior performance is rewarded.

4 Conclusions

In this study, a mean-CVaR optimisation model for international portfolio with currency overlay is proposed. The portfolio is integrated with a currency overlay constructed from foreign exchange forwards providing flexibility for hedging and speculating currency exposure. The portfolio structure allows for the simultaneous optimisation of asset allocation and forward positions for portfolio optimality in terms of asset and currency exposure.

In order to generate realistic scenarios for CVaR calculation, the regular-vine copula is employed to model nonlinear dependency of asset returns (RVC). Thus, the proposed optimisation model demonstrates how to deal with a multicurrency portfolio with a nonlinear return distribution. The RVC optimal portfolio characteristics and performances are then compared to those optimized under the assumption of a multivariate normal return distribution (MVN). RVC and MVN optimal allocations significantly differ in terms of FX hedging position (currency overlay). The former generally have higher currency overlay across risk levels thanks to thorough information on extreme events captured by a regular vine copula model. The out-of-sample performances show that RVC outperform MVN portfolios both in risk and return domains. The results prove the merit of modelling scenarios with non-normal return distribution and non-linear dependence structure.

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